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PAR-003-001515

Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

October / November - 2018

Mathematics : MATH - 503 (A)

(Discrete Mathematics & Complex Analysis - I)

(Old Course)

Faculty Code : 003

Subject Code : 001515

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All the questions are compulsory.

(2) Figure to the right indicate full marks of the question.

1 Answer all the questions : **20**

(1) Define : Equivalence relation.

(2) If R is a relation from $A = \{1, 2, 3\}$ to $B = \{2, 5\}$ R is relation $x < y, x \in A, y \in B$ then find R.

(3) If (S_6, D) is lattice then glb {2, 3} = ________.

(4) The maximal element of Poset ({2, 3, 4, 5}, l) = ________.

(5) For the Poset (S_{30}, D) find 10'.

(6) Write the sum of all minterms of n variables $x_1 . x_2 ..... x_n$.

(7) Find the atoms of Boolean algebra $(S_6, *. \oplus, ', 0, 1)$.

(8) For Boolean algebra $a \oplus 1$ = ________.

(9) If $(L, *, \oplus, 0, 1)$ is bounded lattice then $a \oplus 0$ = ________.

(10) If $(L, \leq)$ is a lattice, $a, b \in L, a \leq b \Leftrightarrow a \cap b$ = ________.

(11) Define : Entire function.

(12) State Cauchy Riemann condition in polar form.

(13) State Laplace equation in polar form.

(14) Write imaginary part of $(\cos\theta + i\sin\theta)^3$.

(15) Show that $f(z) = \bar{z}$ is not analytic.

(16) If L is the length of contour C then L = ________

(17) If $C:|z|=1$ then $\int_{c}\frac{z}{z-2}\,dz$ = __________.

(18) If $C:|z-2|=5$ then $\int_{c}\frac{dz}{z-3}$ = __________.

(19) State Liouville's theorem.

(20) If $C:|z|=1$ then $\int_{c}\frac{\cosh z}{z^4}\,dz$ = __________.

2 (a) Attempt any **three** : **6**

(1) Prove that $R=\{(x,y)\,|\,x,y\in Z, x-y$ is divided by $5\}$ is an equivalence relation.

(2) Draw the Hasse diagram of (S_{12},D) where D = relation of division.

(3) If $(L,*,\oplus,0,1)$ is a bounded lattice then prove that (i) $a*1=a$ (ii) $a\oplus 1=1$.

(4) If $(B,*,\oplus,',0,1)$ is a Boolean algebra then prove that $\forall a,b\in B,(a*b)\oplus(a*b')=a$.

(5) Define : Boolean Homomorphism.

(6) Let $(B,*,\oplus,',0,1)$ be a Boolean algebra then prove that $\forall\ a,b\in B, a\le b\Rightarrow a*b'=0$.

(b) Attempt any **three** : **9**

(1) State and prove modular inequality.

(2) Define : Complete lattice, Bounded lattice.

(3) In a distributive lattice $(L,\le)$ prove that $a\cap b=a\cap c$ and $a\cup b=a\cup c\Rightarrow b=c$.

(4) Prove that non zero element a of Boolean algebra $(B,*,\oplus,',0,1)$ is an atom iff $\forall\ x\in B\ a*x=0$ or $a*x=a$.

(5) Obtain sum of product of $2(x_1,x_2,x_3)=x_1\oplus x_2$.

(6) If m_i and m_j be distinct minterms in n variables $x_1,x_2,\ldots..x_n$ then prove that $m_i*m_j=0$.

(c) Attempt any **two** : **10**

(1) State and prove distributive inequality.

(2) Let L_1 be the lattice D_6 = {1, 2, 3, 6} (divisor of 6) and L_2 be the lattice $(P(S), \subseteq)$ where $S = \{a, b\}$. Then show that D_6 is isomorphic to $P(S)$.

(3) Prove that every chain is distributive lattice.

(4) State and prove D'Morgan's law for Boolean algebra.

(5) $(B, *, \oplus, ', 0, 1)$ is Boolean algebra then prove that for any $x_1, x_2 \in B, A(x_1 * x_2) = A(x_1) \cap A(x_2)$.

3 (a) Attempt any **three** : **6**

(1) Prove that $\exp(z)$ is an analytic function.

(2) Show that $e^x \cos y$ is harmonic function.

(3) Define : Harmonic function, Harmonic conjugate function.

(4) Find $\int_0^{2+i} Z^2 dZ$.

(5) Find $\int_c \frac{Z^2}{Z-1} dZ$ where $C : |Z| = 2$.

(6) Find the value of $\int_c \frac{dZ}{Z-3}$ where $C : |Z-2| = \frac{1}{2}$.

(b) Attempt any **three** : **9**

(1) Prove that real and imaginary part of an analytic function is harmonic function.

(2) If $f(z) = e^x (\cos y - i \sin y)$ then prove that $f'(z) = -f(z)$.

(3) Prove $u = \log r$ is harmonic function and find its harmonic conjugate.

(4) Show that $\left| \int_c \frac{z+4}{z^3-1} dz \right| \leq \frac{6\pi}{7}$ where c be the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$.

(5) In usual notation prove that $\left|\int_a^b f(z)\,dz\right| \le \int_a^b |f(z)|\,dz$.

(6) Find $\int_c \frac{\cos Z}{z\left(z^2+8\right)}\,dz$ where c is square from by $x = \pm 2$ to $y = \pm 2$.

(c) Attempt any **two** : **10**

(1) State and prove Cauchy - Riemann condition in polar form for an analytic function.

(2) Prove that an analytic function of a constant modulus is also constant in its domain.

(3) Prove that $f(z) = \begin{cases} \frac{(\bar{z})^2}{z}, & z \ne 0 \\ 0, & z = 0 \end{cases}$

Satisfied Cauchy - Riemann condition at origin however $f(z)$ is not analytic function at origin.

(4) State and prove Cauchy integral formula.

(5) State and prove Liouville's theorem.